2.3b More on Real Zeros of Polynomial Functions

Ex 1: Given the function: $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$
with factors: $(x + 2)$, $(x - 4)$

a. Verify the factors
use synthetic ÷

$$
\begin{array}{r|rrrrr}
-2 & 8 & -14 & -71 & -10 & 24 \\
 & & -16 & 60 & 22 & -24 \\
\hline
 & 8 & -30 & -11 & 12 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
4 & 8 & -30 & -11 & 12 \\
 & & 32 & 8 & -12 \\
\hline
 & 8 & 2 & -3 & 0
\end{array}
$$

b. Find the remaining factors

$8x^2 + 2x - 3$

$(4x + 3)(2x - 1)$

c. Write the complete factorization

$f(x) = (x+2)(x-4)(4x+3)(2x-1)$

d. List all zeros

$x + 2 = 0$ $\quad x - 4 = 0$ $\quad 4x + 3 = 0$ $\quad 2x - 1 = 0$

$\left\{-2,\ 4,\ -\frac{3}{4},\ \frac{1}{2}\right\}$

Rational Zero Test will relate the possible rational zeros of a polynomial having integer coefficients to the leading coefficients and to the constant term of the polynomial.

The Rational Zero Test

If the polynomial

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

has integer coefficients, then every rational zero of f has the form

$$\text{Rational zero} = \frac{p}{q}$$

where p and q have no common factors other than 1, p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.

Hint --> Find all factors of the constant term (p), then all factors of the leading coefficient (q). Write as $\frac{p}{q}$ and start testing!

Ex 2: Use the Rational Zeros Test to list all possible rational zeros of each function: correction

a. $f(x) = 1x^3 - 4x^2 - 4x + 16$ (q = 1, p = 16)

factors of $p = \pm 1, \pm 2, \pm 4, \pm 16$, ± 8

factors of $q = \pm 1$

$\frac{p}{q} = \boxed{\pm 1, \pm 2, \pm 4, \pm 16}$, ± 8

b. $g(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$ (q = 4, p = 2)

factors of $p = \pm 1, \pm 2$

factors of $q = \pm 1, \pm 2, \pm 4$

$\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{4}$ so

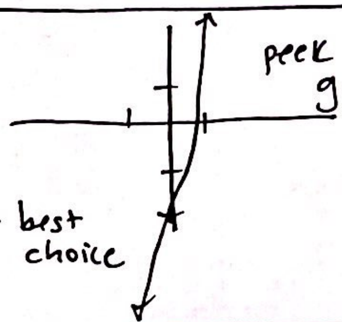
$\frac{p}{q} = \boxed{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2}$

Ex 3: Find all zeros of $f(x) = 6x^3 - 4x^2 + 3x - 2$ (q = 6, p = 2)

$$\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$= \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3}, \pm 2$$

peek at graph



Look at graph, 1 zero, test the choices between 0 and 1, so test $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ and $\frac{2}{3}$ ← best choice

$\frac{2}{3}$	6	−4	3	−2
		4	0	2
	6	0	3	0

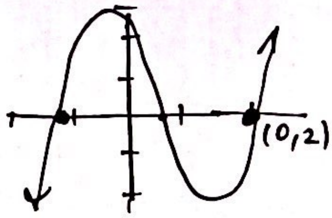
→ $6x^2 + 0x + 3$

$\boxed{f(x) = (x - \frac{2}{3})(6x^2 + 3)}$

Ex 4: Find all real zeros of $f(x) = 10x^4 - 15x^3 - 16x^2 + 12x$

$f(x) = x(10x^3 - 15x^2 - 16x + 12)$

zero is a factor (x); q = 10, p = 12



$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 5, \pm 10}$$

Look at graph and use 2 as a factor

2	10	−15	−16	12
		20	10	−12
	10	5	−6	0

$0 = 10x^2 + 5x - 6$ → Use quad formula to get $x = \frac{-5 \pm \sqrt{265}}{20}$

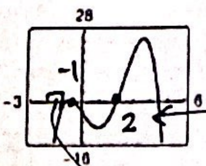
All zeros:

$\{0, 2, \frac{-5 \pm \sqrt{265}}{20}\}$

$\{0, 2, 0.56, -1.06\}$

Ex 5: Given the graph of $y = f(x)$ below, find the zeros. Use a graphing calculator.

$y = -x^4 + 5x^3 - 10x - 4$



$y = -x^4 + 5x^3 - 10x - 4$

Test

2	−1	5	0	−10	−4
		−2	6	12	4
	−1	3	6	2	0 ✓

Test

−1	−1	3	6	2
		1	−4	−2
	−1	4	2	0

→ $0 = -x^2 + 4x + 2$

Use Quad Form

$\{-1, 2, 2 \pm \sqrt{6}\}$