2.5a

p.140 (1, 3-8, 13, 15, 17-41 eoo)

1. Fundamental Theorem of Algebra

3. n linear factors (same as power)

4. $-2i$ because if you have $2i$ then its conjugate pair also comes along with it $\pm 2i$

5. $f(x) = -2x^4 + 32$ (c.) 4 zeros

6. $f(x) = x^5 - x^3$ (d.) 5 zeros

7. $f(x) = x^3 + 3x^2 + 2x$ (b) 3 zeros

8. $f(x) = x^1 - 14$ (a) 1 zero

13. $f(x) = \underbrace{x^3 - 4x^2}\ \underbrace{+x-4}$

Factor by grouping

$f(x) = x^2(x-4) + 1(x-4)$

$f(x) = (x^2+1)(x-4)$

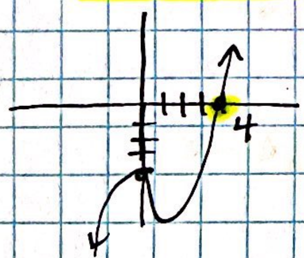
$0 = x^2+1 \qquad 0 = x-4$

$x^2 = -1$

$x = \pm i \qquad x = 4$

zeros: $\pm i$, 4

1 real zero so 1 x-int



15. $f(x) = x^4 + 4x^2 + 4$

Factor

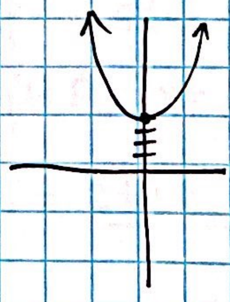
$f(x) = (x^2+2)(x^2+2)$

$0 = x^2+2 \qquad 0 = x^2+2$

$x^2 = -2$ same

$x = \pm i\sqrt{2}$

4 zeros; $\underbrace{-\sqrt{2}i, -\sqrt{2}i}_{\text{double root}}, \underbrace{\sqrt{2}i, \sqrt{2}i}_{\text{double root}}$



No real zeros, so NO x-intercepts

17. $h(x) = x^2 - 4x + 1$

cannot factor, use Q.F.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4}{2} \pm \frac{2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$ All zeros

$$f(x) = (x - (2+\sqrt{3}))(x - (2-\sqrt{3}))$$

$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$ factors

21. $f(x) = x^2 + 25$

cannot factor, set $=0$ and solve

$$0 = x^2 + 25$$

$$-25 = x^2$$

$$x^2 = \pm\sqrt{-25} = \pm 5i$$ 2 zeros (not TR)

$$f(x) = (x - (5i))(x - (-5i))$$

$$= (x - 5i)(x + 5i)$$ factors

25. $f(z) = z^2 - z - 56$

factor

$$0 = (x - 8)(x + 7)$$ factors

$0 = x - 8$ $\quad$ $0 = x + 7$

$x = 8, -7$ zeros

29. $f(x) = 3x^3 - 5x^2 + 48x - 80$

factor by grouping, rearrange

$$f(x) = 3x^3 + 48x - 5x^2 - 80$$

$$= 3x(x^2 + 16) - 5(x^2 + 16)$$

$$0 = (x^2 + 16)(3x - 5)$$

$x^2 + 16 = 0$ $\quad$ $3x - 5 = 0$

$x^2 = -16$ $\quad$ $3x = 5$

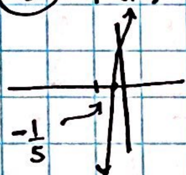
$x = \pm\sqrt{-16}$ $\quad$ $x = 5/3$

$= \pm 4i$

$\pm 4i, \frac{5}{3}$

$$(3x - 5)(x + 4i)(x - 4i)$$

33. $f(x) = 5x^3 - 9x^2 + 28x + 6$



can't factor by grouping → peek at graph, crosses at $-\frac{1}{5}$

now use synthetic ÷

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

$$5x^2 - 10x + 30 = 0$$

$$5(x^2 - 2x + 6) = 0$$

Q.F. $\quad x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(6)}}{2}$

$$= \frac{2 \pm \sqrt{4 - 24}}{2} = \frac{2 \pm \sqrt{-20}}{2}$$

$$= \frac{2}{2} \pm \frac{2i\sqrt{5}}{2} = 1 \pm i\sqrt{5}$$ or $1 \pm \sqrt{5}i$

factors $(x + \frac{1}{5})(x - (1 + \sqrt{5}i))(x - (1 - \sqrt{5}i))$

$$(5x + 1)(x - 1 - \sqrt{5}i)(x - 1 + \sqrt{5}i)$$

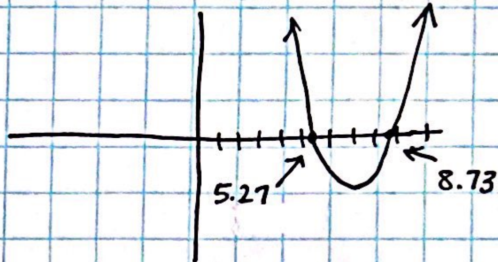
Scanned with CamScanner

(37) $f(x) = x^2 - 14x + 46$

can't factor, use Q.F.

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(46)}}{2(1)} = \frac{14 \pm \sqrt{196 - 184}}{2} = \frac{14 \pm \sqrt{12}}{2}$$

$$= \frac{14}{2} \pm \frac{2\sqrt{3}}{2} = 7 \pm \sqrt{3}$$ (a)



(b) linear factors $(x - (7+\sqrt{3}))(x - (7-\sqrt{3}))$

$(x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$

(c) x-intercepts are located at $(7+\sqrt{3}, 0)$ and $(7-\sqrt{3}, 0)$

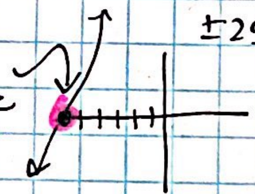
≈ 8.73 ≈ 5.27

---

(41) $f(x) = x^3 - 11x + 150$ (a, p)

can't factor – use $\frac{p}{q}$ to test possible roots $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 5, \pm 25, \pm 30, \pm 50, \pm 75, \pm 150$

or peek at graph and see -6



Use synthetic division to test -6

$$\begin{array}{r|rrrr} -6 & 1 & 0 & -11 & 150 \\ & & -6 & 36 & -150 \\ \hline & 1 & -6 & 25 & 0 \end{array}$$

$1x^2 - 6x + 25$ now use Q.F.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)} = \frac{6 \pm \sqrt{36 - 100}}{2} = \frac{6 \pm \sqrt{-64}}{2}$$

$$= \frac{6}{2} \pm \frac{8i}{2} = 3 \pm 4i$$

(a) $-6, 3 \pm 4i$

(b) $(x-6)(x - (3+4i))(x - (3-4i))$

$(x-6)(x - 3 - 4i)(x - 3 + 4i)$

(c) x-int is only at $(-6, 0)$

Scanned with CamScanner