Pre-Calc 2.5a Notes

The Fundamental Theorem of Algebra

Fundamental Thm of Alg:

If f(x) is a polynomial of degree n, where n > 0, then f has at least one zero in the complex number system.

What does this mean?

If you have a third degree polynomial - it will have 3 zeros (can be real and/or imaginary)and 3 linear factors!

Recall: → Zero = x-intercept = solution

Zero = -2 then factor is (x + 2)

Ex: $n=3$

$f(x) = x^3 - x^2 - 6x$

$= x(x^2 - x - 6)$

$0 = x(x-3)(x+2)$

$x = 0 \quad 0 = x-3 \quad 0 = x-2$

$x = 0, 3, 2$

3 solutions all IR

Complex Zeros Occur in Conjugate Pairs

Let f be a polynomial function that has real coefficients. If a + bi, where b = 0 is a zero of the function, then the conjugate a - bi is also a zero of the function.

Complex zeros always travel in pairs

Ex:

- $5 + i$ and $5 - i$
- $3 - \sqrt{5}$ and $3 + \sqrt{5}$

Ex: if $-2 - 3i$ is a zero, then so is $-2 + 3i$. (same, opposite)

You TRY: If $4 + 2i$ is a zero, then so is $4 - 2i$ (same, opp)

Look at the power!!

Example 1: Match the function with its exact # of zeros.

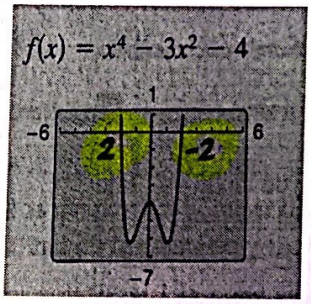
(a) 1 zero — $f(x) = x^2 - 14x = x(x-14)$

(b) 2 zeros — $f(x) = 3x^4 - 2x - 1$

(c) 3 zeros — $f(x) = (x+2)$

(d) 4 zeros — $f(x) = -2x^3 - 3x^2 + 4x - 5$

Example 2: Find all zeros of the function. What is the relationship between the number of real zeros and the number of x-intercepts of the graph?



$f(x) = x^4 - 3x^2 - 4$

$0 = (x^2 - 4)(x^2 + 1)$

$x^2 - 4 = 0$ | $x^2 + 1 = 0$

$x^2 = 4$ | $x^2 = -1$

$x = \pm 2$ real | $x = \pm i$ imaginary

There are 2 real solutions, the real solutions are also x-intercepts. Imaginary solutions do not exist on x-axis

Example 3: Find all the zeros of the function and write the polynomial as a product of linear factors.

(a) $f(x) = x^2 + 10x + 23$ cannot factor, so QF.

$= \frac{-10 \pm \sqrt{10^2 - 4(1)(23)}}{2(1)} = \frac{-10 \pm \sqrt{100 - 92}}{2}$

$\frac{10 \pm \sqrt{8}}{2} = \frac{-10}{2} \pm \frac{2\sqrt{2}}{2} = -5 \pm \sqrt{2}$ 2 zeros

product of factors

$f(x) = (x - (-5+\sqrt{2}))(x - (-5-\sqrt{2}))$

$f(x) = (x + 5 - \sqrt{2})(x + 5 + \sqrt{2})$

(b) $f(x) = 3x^3 - 2x^2 + 75x - 50$ rearrange to group

* 4 terms, try grouping!

$f(x) = \underline{3x^3 + 75x} \; \underline{-2x^2 - 50}$

$= 3x(x^2 + 25) - 2(x^2 + 25)$

$0 = (3x - 2)(x^2 + 25)$

$3x - 2 = 0$ | $x^2 + 25 = 0$

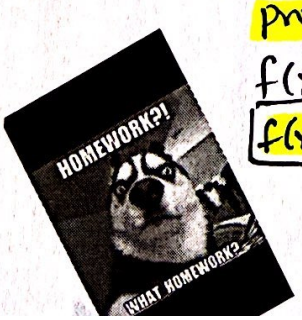
$x = \frac{2}{3}$ | $x^2 = -25$

$x = \pm 5i$

3 zeros; $\frac{2}{3}, \pm 5i$

product of factors

$f(x) = (x - \frac{2}{3})(x - 5i)(x + 5i)$



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