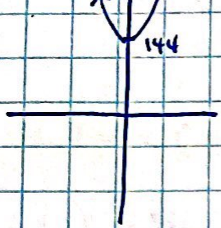
2.5b

p.140-141 (43-53 odd, 59-65 odd, 69)

(43.) $f(x) = x^4 + 25x^2 + 144$ Factor



no real roots

$x^4 + 25x^2 + 144 = 0$

$(x^2+9)(x^2+16) = 0$

$x^2+9=0$ $\quad$ $x^2+16=0$

$x^2 = -9$ $\quad$ $x^2 = -16$

$x = \pm 3i$ $\quad$ $x = \pm 4i$

(a) $\pm 3i, \pm 4i$

(b) $(x+4i)(x-4i)(x+3i)(x-3i)$

(c) no real roots

(45.) zeros: $5, i, -i$

$f(x) = (x-5)(x-i)(x-(-i))$

$= (x-5)\underbrace{(x-i)(x+i)}_{\text{FOIL}}$

$= (x-5)(x^2 \cancel{+xi} \cancel{-xi} - i^2)$

$= (x-5)(x^2 - (-1))$

$= (x-5)(x^2+1)$

FOIL

$f(x) = x^3 + x - 5x^2 - 5$

$f(x) = x^3 - 5x^2 + x - 5$

(47.) zeros: $1, 1, 2i, -2i$

$(x+2i)$

$f(x) = \underbrace{(x-1)(x-1)}_{\text{FOIL}}\underbrace{(x-2i)(x-(-2i))}_{\text{FOIL}}$

$= (x^2-2x+1)(x^2 \cancel{+2xi} \cancel{-2xi} - 4i^2)$

$= (x^2-2x+1)(x^2+4)$

$= x^4 + 4x^2 - 2x^3 - 8x + x^2 + 4$

$f(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$

(49.) $0, -5, 1+\sqrt{2}i, 1-\sqrt{2}i$

$f(x) = (x-0)(x+5)(x-(1+\sqrt{2}i))(x-(1-\sqrt{2}i))$

$= \underbrace{x(x+5)}\underbrace{(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)}$

$= (x^2+5x)((x-1)-\sqrt{2}i)((x-1)+\sqrt{2}i)$

$= (x^2+5x)[(x-1)^2 - \sqrt{4}i^2]$

$= (x^2+5x)[x^2-2x+1+2]$

$= (x^2+5x)(x^2-2x+3)$

$= x^4 - 2x^3 + 3x^2 + 5x^3 - 10x^2 + 15x$

$f(x) = x^4 + 3x^3 - 7x^2 + 15x$

(51.) Degree 4 | Zeros $-1, 2, i, -i$ | Solution Point $f(1) = 8$

$$f(x) = a(x+1)(x-2)(x-i)(x+i)$$
$$= a(x^2 - x - 2)(x^2 - i^2)$$
$$= a(x^2 - x - 2)(x^2 + 1)$$
$$= a(x^4 + x^2 - x^3 - x - 2x^2 - 2)$$
$$f(x) = a(x^4 - x^3 - x^2 - x - 2)$$

Since
$$f(1) = 8$$
$$f(1) = a(1^4 - 1^3 - 1^2 - 1 - 2) = 8$$
$$-4a = 8$$
$$a = -2$$

$$f(x) = -2(x^4 - x^3 - x^2 - x - 2)$$

(b) $f(x) = -2x^4 + 2x^3 + 2x^2 + 2x + 4$

(a) $f(x) = -2(x+1)(x-2)(x-i)(x+i)$

(53.) Degree 3 | Zeros $-1, 2+\sqrt{5}i, 2-\sqrt{5}i$ | Sol Point $f(2) = 45$

$$f(x) = a(x+1)(x-(2+\sqrt{5}i))(x-(2-\sqrt{5}i))$$
$$= a(x+1)((x-2)-\sqrt{5}i)((x-2)+\sqrt{5}i)$$
$$= a(x+1)((x-2)^2 - 5i^2)$$
$$= a(x+1)(x^2 - 4x + 4 + 5)$$
$$= a(x+1)(x^2 - 4x + 9)$$
$$= a(x^3 - 4x^2 + 9x + x^2 - 4x + 9)$$
$$= a(x^3 - 3x^2 + 5x + 9)$$

$$f(2) = a(2^3 - 3(2)^2 + 5(2) + 9) = 45$$
$$a(8 - 12 + 10 + 9) = 45$$
$$15a = 45$$
$$a = 3$$

$$f(x) = 3(x^3 - 3x^2 + 5x + 9)$$

(b) $f(x) = 3x^3 - 9x^2 + 15x + 27$

(a) $3(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

(59) $f(x) = 2x^3 + 3x^2 + 50x + 75$ | Zero $5i$

zeros $5i$ and $-5i$

conj pair

$$(x-5i)(x+5i)$$
$$x^2 - 25i^2 = x^2 + 25$$

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2 + 0x + 25 & 2x^3 + 3x^2 + 50x + 75 \\ & -2x^3 - 0x^2 - 50x \\ \hline & 3x^2 + 0x + 75 \\ & -3x^2 - 0x - 75 \\ \hline & 0 \end{array}$$

Factors $(2x+3)(x^2+25)$

Zeros $-\frac{3}{2}, 5i, -5i$

(61) $g(x) = x^3 - 7x^2 - x + 87$ | Zero $5+2i$

Zeros $5+2i$ and $5-2i$

$$(x-(5+2i))(x-(5-2i))$$
$$[(x-5)-2i][(x-5)+2i]$$
$$(x-5)^2 - 4i^2$$
$$x^2 - 10x + 25 + 4$$
$$x^2 - 10x + 29$$

$$\begin{array}{r|l} & x + 3 \\ \hline x^2 - 10x + 29 & x^3 - 7x^2 - x + 87 \\ & -x^3 + 10x^2 - 29x \\ \hline & 3x^2 - 30x + 87 \\ & -3x^2 + 30x - 87 \\ \hline & 0 \end{array}$$

Factors $(x+3)(x^2-10x+29)$

zeros $-3, 5 \pm 2i$

Scanned with CamScanner

63. $h(x) = 3x^3 - 4x^2 + 8x + 8$

Zero

$1-\sqrt{3}i, \; 1+\sqrt{3}i$

$(x-(1-\sqrt{3}i))(x-(1+\sqrt{3}i))$

$((x-1)+\sqrt{3}i)((x-1)-\sqrt{3}i)$

$(x-1)^2 - 3i^2$

$x^2-2x+1+3$

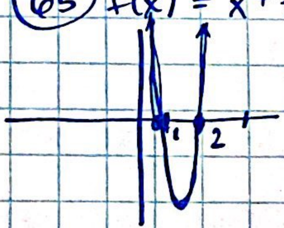
x^2-2x+4

$$\begin{array}{r|l} & 3x + 2 \\ x^2-2x+4 & 3x^3 - 4x^2 + 8x + 8 \\ & -3x^3 + 6x^2 - 12x \\ \hline & \qquad 2x^2 - 4x + 8 \\ & \quad -2x^2 + 4x - 8 \\ \hline & \qquad\qquad 0 \end{array}$$

Factors $(3x+2)(x^2-2x+4)$

Zeros $-\frac{2}{3}, \; 1 \pm \sqrt{3}i$

65. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$



a) real zeros: 1, 2

b)

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -5 & -21 & 22 \\ & & 1 & 4 & -1 & -22 \\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

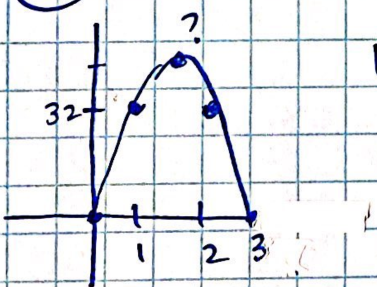
$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -22 \\ & & 2 & 12 & -22 \\ \hline & 1 & 6 & 11 & 0 \end{array}$$

$x^2 + 6x + 11 = 0$

$x = \frac{-6 \pm \sqrt{36 - 4(1)(11)}}{2} = \frac{-6 \pm \sqrt{-8}}{2}$

$= -\frac{6}{2} \pm \frac{2\sqrt{2}i}{2} = -3 \pm \sqrt{2}i$

69. $h(t) = -16t^2 + 48t \qquad 0 \le t \le 3$



let the height = 50

$50 = -16t^2 + 48t$

$0 = -16t^2 + 48t - 50$

$x = \frac{-48 \pm \sqrt{48^2 - 4(-16)(-50)}}{2(-16)} = \frac{-48 \pm \sqrt{2304 - 3200}}{-32}$

$= \frac{-48 \pm \sqrt{-896}}{-32}$

imaginary root so the football never reaches a real #