Pre-Calc 2.5b Notes

Fundamental Theorem of Algebra Part II

Recall → If $x = 4$ is a zero, then $(x - 4)$ is the factor

If $x = 1+2i$ is a zero, then $1-2i$ is also a zero

Recall Write the linear factorization for the given zeros: -2, 3, 2i

$$\boxed{(x+2)(x-3)(x-2i)(x+2i)}$$

Linear factorization	$f(x) = a_n(x-c_1)(x-c_2)(x-c_3)\ldots(x-c_n)$	n linear factors
Poly equation	$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$	power of n

Example 1: Find a polynomial function $f(x)$ with real coefficients that has the given zeros.

3, 4i, -4i

$a(x-3)\underbrace{(x-4i)(x+4i)}_{\text{mult. here first FOIL}}$

$f(x) = a(x-3)(x^2 + \cancel{4xi} - \cancel{4xi} - 16i^2)$

$i^2 = (-1)$

$= a(x-3)(x^2 - 16(-1))$

$= a(x-3)(x^2 + 16)$

FOIL again

let $a = 1$

$a(x^3 + 16x - 3x^2 - 48)$

$f(x) = x^3 - 3x^2 + 16x - 48$

Example 2: Write a polynomial function that has the given information.

conjugate $-\sqrt{3}i$; apply at the end

Degree: 4   Zeros: 1, 4, √3i,   Solution point: f(0)= -6

$f(x) = a\underbrace{(x-1)(x-4)}_{\text{FOIL}}\underbrace{(x-\sqrt{3}i)(x+\sqrt{3}i)}_{\text{FOIL}}$ $i^2 = -1$

$= a(x^2-4x-1x+4)(x^2+\cancel{\sqrt{3}xi}-\cancel{\sqrt{3}xi}-\sqrt{9}i^2)$

$\underbrace{a(x^2-5x+4)(x^2+3)}_{\text{multiply}}$ 1

$= a(x^2-5x+4)(x^2+3)$

$= a(x^4+3x^2-5x^3-15x+4x^2+12)$

$f(x) = a(x^4-5x^3+7x^2-15x+12)$

GOOD STUFF

$f(0) = -6$

$f(0) = a((0)^3-5(0)^3+7(0)^2-15(0)+12) = -6$

$a(0-0+0-0+12) = -6$

$12a = -6$

$a = -\frac{1}{2}$

So

$= -\frac{1}{2}(x^4-5x^3+7x^2-15x+12)$

$f(x) = -\frac{1}{2}x^4 + \frac{5}{2}x^3 - \frac{7}{2}x^2 + \frac{15}{2}x - 6$

and

$-\frac{1}{2}(x-1)(x-4)(x-\sqrt{3}i)(x+\sqrt{3}i)$

★

Example 3: Use the given zero to find all the zeros of the function.

$$f(x)=4x^3+23x^2+34x-10$$

zero: $\underbrace{-3+i}$, $-3-i$

must have a conjugate pair

use these zeros to set up factors

$(x-(-3+i))(x-(-3-i))$

$(x+3-i)(x+3+i)$

mult out or mult with grouping

$+3x+\cancel{xi}+3x+9+\cancel{3i}-\cancel{xi}-\cancel{3i}-i^2$

$x^2+6x+9+1$

$x^2+6x+10$

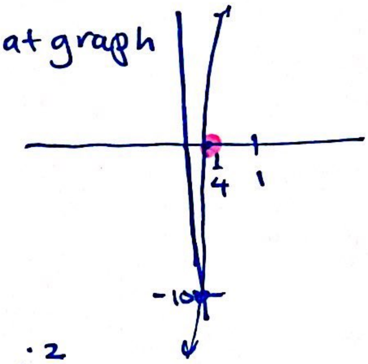
$[(x+3)-i][(x+3)+i]$

$(x+3)^2+\cancel{(x+3)i}-\cancel{(x+3)i}-i^2$

$x^2+6x+9-(-1)$

$x^2+6x+10$

peek at graph



ow use long division, divide $x^2+6x+10$ into $4x^3+23x^2+34x-10$

$$\begin{array}{r|l} & 4x-1 \\ \hline x^2+6x+10 & 4x^3+23x^2+34x-10 \\ & -4x^3-24x^2-40x \\ \hline & -x^2-6x-10 \\ & +x^2+6x+10 \\ \hline & 0 \end{array}$$

0 ← shows we found the factor

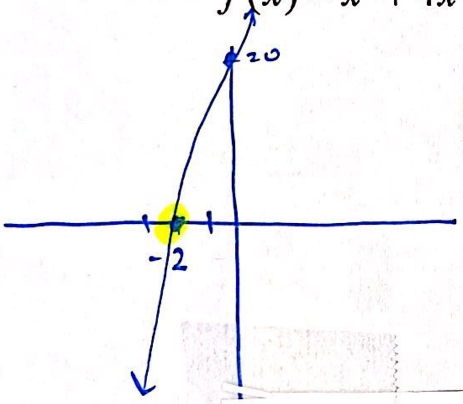
$4x-1=0$

$x=\frac{1}{4}$

Zeros: $\frac{1}{4}$, $-3+i$, $-3-i$

Example 4: Use a graphing utility to find the real zeros of the function, and then use the real zeros to find the exact values of the imaginary roots.

$$f(x) = x^3 + 4x^2 + 14x + 20$$



(a) real zero is located at -2

(b.) use synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 14 & 20 \\ & & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

$x^2 + 2x + 10$

use Q.F. $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)} = \dfrac{-2 \pm \sqrt{4 - 40}}{2}$

$= \dfrac{-2 \pm \sqrt{-36}}{2} = \dfrac{-2}{2} \pm \dfrac{6i}{2} = -1 \pm 3i$

imaginary roots

Hmk → p. 140–141 → (43–53 odd, 59–65 odd, 69)