3.3b Condensing Logarithmic Expressions

Remember these properties of Logarithms? Fill in the blanks

$\log_a x + \log_a y = \log_a \boxed{xy}$

$x \log_a y = \log_a \boxed{y^x}$

$\log_a x - \log_a y = \log_a \boxed{\frac{x}{y}}$

try to write as a single logarithm

Ex 1: Use the properties of logarithms to condense each expression:

a. $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$ $= \frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$ $= \log_{10} x^{\frac{1}{2}} + \log_{10}(x+1)^3$ $= \log_{10} x^{\frac{1}{2}} \cdot (x+1)^3$ $= \log_{10} \sqrt{x}\,(x+1)^3$	b. $2\ln(x+2) - \ln x$ $= 2\ln(x+2) - \ln x$ $= \ln(x+2)^2 - \ln x$ $= \ln \frac{(x+2)^2}{x}$
c. $\log_5 8 - \log_5 t$ $= \log_5 \frac{8}{t}$	d. $\ln(x-2) + \ln 2 - 3\ln y$ $= \ln(x-2) + \ln 2 - \ln y^3$ $= \ln \frac{(x-2)2}{y^3}$ $= \ln \frac{2(x-2)}{y^3}$
e. $\frac{1}{3}[\log_2 x + \log_2(x-4)]$ $= \frac{1}{3}[\log_2 x(x-4)]$ $= \log_2 [x(x-4)]^{\frac{1}{3}}$ $= \log_2 \sqrt[3]{x(x-4)}$	f. $4[\ln z + \ln(z+5)] - 2\ln(x-5)$ $= 4[\ln z(z+5)] - 2\ln(x-5)$ $= \ln[z(z+5)]^4 - \ln(x-5)^2$ $= \ln \frac{(z(z+5))^4}{(x-5)^2} = \ln \frac{z^4(z+5)^4}{(x-5)^2}$

Ex 2: Find the exact value of the logarithm without using a calculator:

a. $\log_6 6$ $= \boxed{1}$ $\log_a a = 1$	b. $\log_5\left(\frac{1}{125}\right)$ $\log_5 \frac{1}{5^3}$ $\log_5 5^{-3} = -3(\log_5 5)$ $= -3(1) = \boxed{-3}$
c. $\log_4 2 + \log_4 32$ $= \log_4(2 \cdot 32)$ $= \log_4(64)$ $= \log_4 (4)^3$ $= 3(\log_4 4)$ $= 3(1)$ $= \boxed{3}$	d. $\ln \sqrt[5]{e^3}$ $= \log_e e^{\frac{3}{5}}$ $= \frac{3}{5}(\log_e e)$ $= \frac{3}{5}(1) = \boxed{\frac{3}{5}}$
e. $3 \ln e^5$ $= 3(\log_e e^5)$ $= 3(5 \log_e e)$ $= 3(5)(1) = \boxed{15}$	f. $\ln e^6 - 2 \ln e^7$ $= 6(\log_e e) - 2(7)[\log_e e]$ $= 6(1) - 14(1)$ $6 - 14$ $\boxed{-8}$

Ex 3: True or False

True a. $\log 10 = 1$ $\quad \log_{10} 10 = 1 \quad 10^1 = 10$ ✓

False b. $\log_3 8 = 2$ $\quad 3^2 \neq 8$

True c. $\ln \frac{1}{e^2} = -2$ $\quad \log_e \frac{1}{e^2} = -2 \quad \log_e e^{-2} = -2 \quad e^{-2} = e^{-2}$

False d. $\ln e = 0$ $\quad \log_e e = 1$

True e. $\frac{\log_{10} 5}{\log_{10} 2} = \frac{\ln 5}{\ln 2}$

True f. $\log_{10} \frac{1}{100} = -2$ $\quad \log_{10} \frac{1}{10^2} = -2 \quad \log_{10} 10^{-2} = -2$

$10^{-2} = 10^{-2}$

True g. $\log_e x = \ln x$

False h. $\log_g b = \frac{\log_{10} g}{\log_{10} b}$