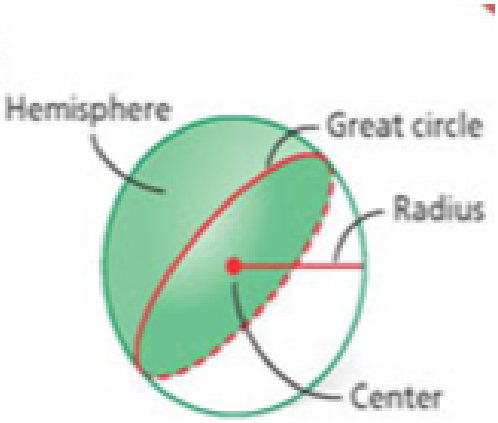
11-4 Spheres

A _____ is the locus of points in space that are a fixed distance form a given point called the _____ _____.

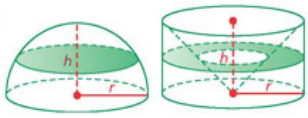
A _____ is half of a sphere.

A _____ divides a sphere into two heimspheres.



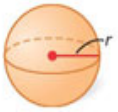
The figure shows a hemisphere and a cylinder with a cone removed from its interior. The cross sections have the same area at every level, so the volumes are equal by Cavalieri's Principle. You will prove that the cross sections have equal areas in Exercise 39.

$$\begin{aligned} V(\text{hemisphere}) &= V(\text{cylinder}) - V(\text{cone}) \\ &= \pi r^2 h - \frac{1}{3}\pi r^2 h \\ &= \frac{2}{3}\pi r^2 h \\ &= \frac{2}{3}\pi r^2 (r) \quad \textit{The height of the hemisphere is equal to the radius.} \\ &= \frac{2}{3}\pi r^3 \end{aligned}$$



Volume of a Sphere

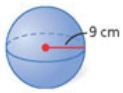
The volume of a sphere with radius r is



1 Finding Volumes of Spheres

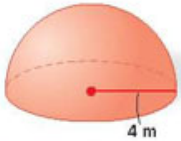
Find each measurement. Give your answer in terms of π.

A the volume of the sphere



B the diameter of a sphere with volume 972π in^3

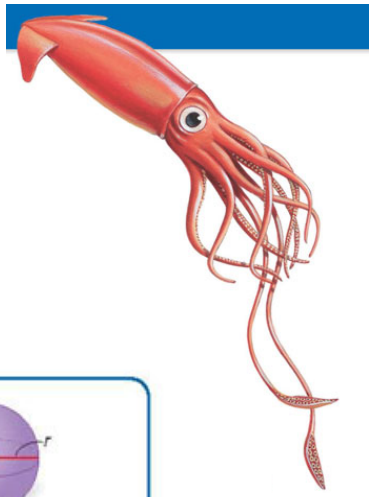
C the volume of the hemisphere



2 Biology Application

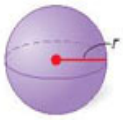
Giant squid need large eyes to see their prey in low light. The eyeball of a giant squid is approximately a sphere with a diameter of 25 cm, which is bigger than a soccer ball. A human eyeball is approximately a sphere with a diameter of 2.5 cm. How many times as great is the volume of a giant squid eyeball as the volume of a human eyeball?

human eyeball: | giant squid eyeball:



Surface Area of a Sphere

The surface area of a sphere with radius r is $S =$



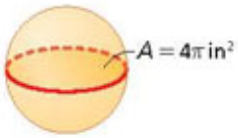
3 Finding Surface Area of Spheres

Find each measurement. Give your answers in terms of π.

A the surface area of a sphere with diameter 10 ft

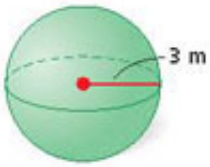
B the volume of a sphere with surface area 144π m^2

C the surface area of a sphere with a great circle that has an area of 4π in^2



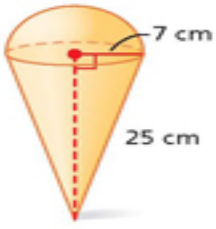
4 Exploring Effects of Changing Dimensions

The radius of the sphere is tripled. Describe the effect on the volume.



5 Finding Surface Areas and Volumes of Composite Figures

Find the surface area and volume of the composite figure. Give your answers in terms of π.

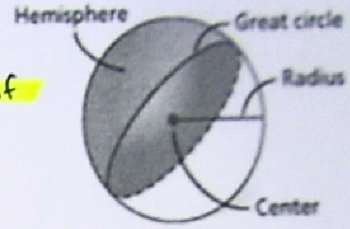


11-4 Spheres

A sphere is the locus of points in space that are a fixed distance form a given point called the center of the sphere.

A hemisphere is half of a sphere.

A great circle divides a sphere into two heimspheres.



The figure shows a hemisphere and a cylinder with a cone removed from its interior. The cross sections have the same area at every level, so the volumes are equal by Cavalieri's Principle. You will prove that the cross sections have equal areas in Exercise 39.

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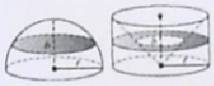


Volume of hemisphere $V = \frac{2\pi r^3}{3}$

Now double to get sphere $V = 2\left(\frac{2\pi r^3}{3}\right)$

Volume of a Sphere

The volume of a sphere with radius r is

$$V = \frac{4\pi r^3}{3}$$

1 Finding Volumes of Spheres

Find each measurement. Give your answer in terms of π.

A the volume of the sphere

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(9)^3}{3} = \frac{4\pi(729)}{3}$$

$$= 972\pi \text{ cm}^3$$

B the diameter of a sphere with volume 972π in^3

$$V = \frac{4\pi r^3}{3}$$

$$\frac{972\pi}{1} = \frac{4\pi r^3}{3}$$

$$\frac{2916\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$729 = r^3$$

Now take the cube root $\sqrt[3]{\ }$ or raise to $\frac{1}{3}$ power

$$\sqrt[3]{729} = r$$

$$r = 9$$

diameter = 18 in

C the volume of the hemisphere

$$V = \frac{2\pi r^3}{3}$$

$$= \frac{2\pi(4)^3}{3} = \frac{128\pi}{3} \text{ in}^3$$

$V = \frac{4\pi r^3}{3}$

2 Biology Application

Giant squid need large eyes to see their prey in low light. The eyeball of a giant squid is approximately a sphere with a diameter of 25 cm, which is bigger than a soccer ball. A human eyeball is approximately a sphere with a diameter of 2.5 cm. How many times as great is the volume of a giant squid eyeball as the volume of a human eyeball?

human eyeball: $d = 2.5$ cm, $r = 1.25$ cm

$V = \frac{4\pi(1.25)^3}{3} \approx 8.18 \text{ cm}^3$

giant squid eyeball: $d = 25$ cm, $r = 12.5$ cm

$V = \frac{4\pi(12.5)^3}{3} \approx 8181.23 \text{ cm}^3$

$\approx \frac{8181.23}{8.18}$

$\approx$ 1000 times as great

if you take a basketball apart and look at its surface area you will see

4 equal circles

Surface Area of a Sphere

The surface area of a sphere with radius r is $S =$

$SA = 4\pi r^2$

3 Finding Surface Areas of Spheres

Find each measurement. Give your answers in terms of π.

A the surface area of a sphere with diameter 10 ft

$r = 5$ ft

$SA = 4\pi r^2$

$= 4\pi(5)^2$

$= 4\pi(25) = 100\pi \text{ ft}^2$

B the volume of a sphere with surface area $144\pi \text{ m}^2$

$SA = 144\pi$

$4\pi r^2 = 144\pi$

$\frac{4\pi r^2}{4\pi} = \frac{144\pi}{4\pi}$

$r^2 = 36$

$r = 6$

$V = \frac{4\pi r^3}{3}$

$= \frac{4\pi(6)^3}{3} = 288\pi \text{ m}^3$

C the surface area of a sphere with a great circle that has an area of $4\pi \text{ in}^2$

A great circle has Area $\pi r^2 = 4\pi$

$r^2 = 4$, $r = 2$

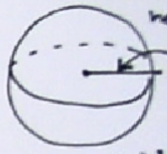
$SA = 4\pi r^2$

$= 4\pi(2)^2$

$= 16\pi \text{ in}^2$

4 Exploring Effects of Changing Dimensions

The radius of the sphere is tripled. Describe the effect on the volume.



$V = \frac{4\pi(3)^3}{3}$

$= 36\pi$

$V = \frac{4\pi(9)^3}{3}$

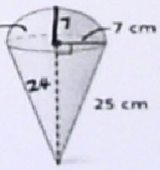
$= 972\pi$

$\frac{972\pi}{36\pi} = 27$ times as large

or $(3)^3$

5 Finding Surface Areas and Volumes of Composite Figures

Find the surface area and volume of the composite figure. Give your answers in terms of π.



hemisphere

$SA = 2\pi r^2$

$= 2\pi(7)^2$

$= 98\pi \text{ cm}^2$

$V = \frac{2\pi r^3}{3} = \frac{2\pi(7)^3}{3}$

$= \frac{686\pi}{3} \text{ cm}^3$

cone

$SA = \pi r l$

$\pi(7)(25)$

$175\pi \text{ cm}^2$

$V = \frac{\pi r^2 h}{3} = \frac{\pi(7)^2(24)}{3} = 392\pi \text{ cm}^3$

Total

$SA = 98\pi + 175\pi = 273\pi \text{ cm}^2$

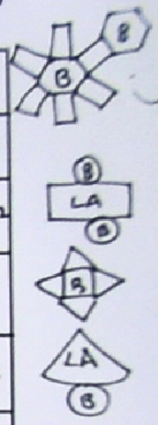
$V = \frac{686\pi}{3} + 392\pi = \frac{1862\pi}{3} \text{ cm}^3$

Area	
Square	$A = s^2$
Rectangle	$A = bh$
Parallelogram	$A = bh$
Trapezoid	$A = \frac{h(b_1+b_2)}{2}$
Kite/Rhombus	$A = \frac{d_1 \cdot d_2}{2}$
Triangle	$A = \frac{bh}{2}$ $A = \frac{1}{2}ab\sin C$
Regular Polygon	$A = \frac{ap}{2}$

Equilateral △ $A = \frac{s^2\sqrt{3}}{4}$

Extra Info

	V	LA	SA
	Volume	Lateral Area	Surface Area
Prism	$V = Bh$	$LA = p \cdot h$ (perimeter of base) height	$LA + 2B$
Cylinder	$V = \pi r^2 h$	$LA = 2\pi r h$	$LA + 2\pi r^2$
Pyramid	$V = \frac{Bh}{3}$	$LA = \frac{p\ell}{2}$	$LA + B$
Cone	$V = \frac{\pi r^2 h}{3}$	$LA = \pi r \ell$	$LA + \pi r^2$
Sphere	$V = \frac{4\pi r^3}{3}$	✗	$SA = 4\pi r^2$
Hemisphere	$V = \frac{2\pi r^3}{3}$	✗	$SA = 2\pi r^2$



ℓ = slant height, height of a face