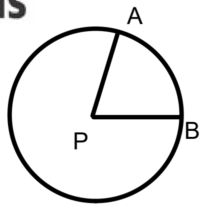
# 12-2 Arcs and Chords

A ________ is an angle whose vertex is the center of a circle. An____ is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

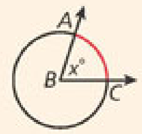
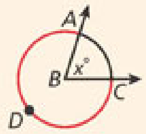
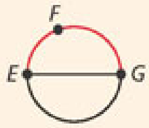



OP

central angle____

intercepted arc ___

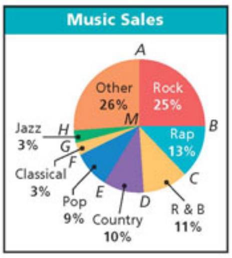
## Arcs and Their Measure

| ARC | MEASURE | DIAGRAM |
|---|---|---|
| A ________ is an arc whose points are on or in the interior of a central angle. | The measure of a minor arc is equal to the measure of its central angle. $m\overset{\frown}{AC} = m\angle ABC = x°$ |  |
| A ________ is an arc whose points are on or in the exterior of a central angle. | The measure of a major arc is equal to 360° minus the measure of its central angle. $m\overset{\frown}{ADC} = 360° - m\angle ABC = 360° - x°$ |  |
| If the endpoints of an arc lie on a diameter, the arc is a ________. | The measure of a semicircle is equal to 180°. $m\overset{\frown}{EFG} = 180°$ |  |

### 1 *Data Application*

The circle graph shows the types of music sold during one week at a music store. Find $m\overset{\frown}{BC}$.

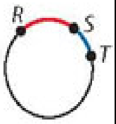
$m\overset{\frown}{BC} = m\angle BMC$   *m of arc = m of*




CHECK IT OUT! Use the graph to find each of the following.

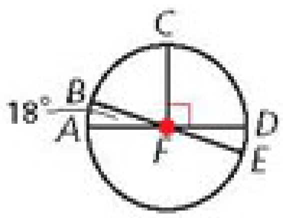
**1a.** $m\angle FMC$   **1b.** $m\overset{\frown}{AHB}$   **1c.** $m\angle EMD$

________ are arcs of the same circle that intersect at exactly one point. $\overset{\frown}{RS}$ and $\overset{\frown}{ST}$ are adjacent arcs.



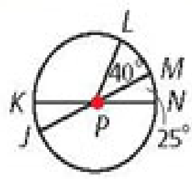

### 2 Using the Arc Addition Postulate

Find $m\overset{\frown}{CDE}$

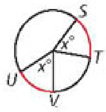



Find each measure.

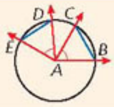
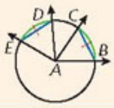
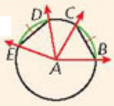
**2a.** $m\overset{\frown}{JKL}$   **2b.** $m\overset{\frown}{LJN}$



Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure, $\widehat{ST} \cong \widehat{UV}$.



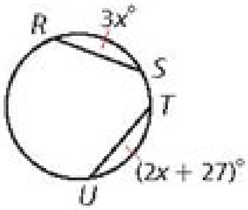

**Theorem 12-2-2**

| THEOREM | HYPOTHESIS | | CONCLUSION |
|---|---|---|---|
| In a circle or congruent circles: | | | |
| (1) Congruent central angles have _______ _______ |  | $\angle EAD \cong \angle BAC$ | $\overline{DE} \cong \overline{BC}$ |
| (2) Congruent chords have _______ _______ |  | $\overline{ED} \cong \overline{BC}$ | $\widehat{DE} \cong \widehat{BC}$ |
| (3) Congruent arcs have congruent _______ _______ |  | $\widehat{ED} \cong \widehat{BC}$ | $\angle DAE \cong \angle BAC$ |

## 3 Applying Congruent Angles, Arcs, and Chords

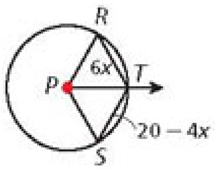
Find each measure.

a **A** $\overline{RS} \cong \overline{TU}$. Find $m\widehat{RS}$.

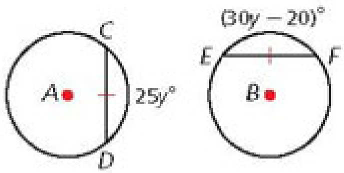



Find each measure.

**3a.** $\overrightarrow{PT}$ bisects $\angle RPS$. Find $RT$.

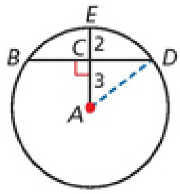



**3b.** $\odot A \cong \odot B$, and $\overline{CD} \cong \overline{EF}$. Find $m\widehat{CD}$.

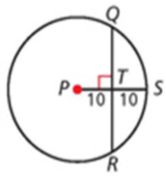



## 4 Using Radii and Chords

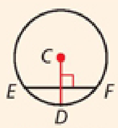
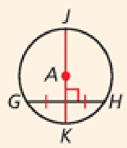
Find $BD$.




**4.** Find $QR$ to the nearest tenth.

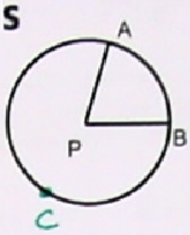



**Theorems**

| THEOREM | | HYPOTHESIS | CONCLUSION |
|---|---|---|---|
| **12-2-3** | In a circle, if a radius (or diameter) is perpendicular to a chord, then it _______ the chord and its arc. |  $\overline{CD} \perp \overline{EF}$ | $\overline{CD}$ bisects $\overline{EF}$ and $\widehat{EF}$. |
| **12-2-4** | In a circle, the perpendicular bisector of a chord is a _______ (or _______). |  $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$. | $\overline{JK}$ is a diameter of $\odot A$. |

# 12-2 Arcs and Chords

A central ∠ is an angle whose vertex is the center of a circle. An arc is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.



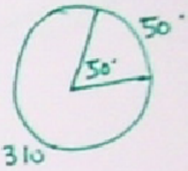

⊙P

central angle ∠APB

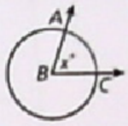
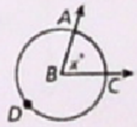
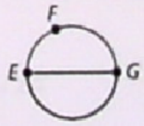
intercepts arc $\widehat{AB}$ ← minor

intercepted

$\widehat{ACB}$ ← major




## Arcs and Their Measure

| ARC | MEASURE | DIAGRAM |
|---|---|---|
| A minor arc is an arc whose points are on or in the interior of a central angle. | The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x°$ |  |
| A major arc is an arc whose points are on or in the exterior of a central angle. | The measure of a major arc is equal to 360° minus the measure of its central angle. $m\widehat{ADC} = 360° - m\angle ABC = 360° - x°$ |  |
| If the endpoints of an arc lie on a diameter, the arc is a semicircle | The measure of a semicircle is equal to 180°. $m\widehat{EFG} = 180°$ |  |

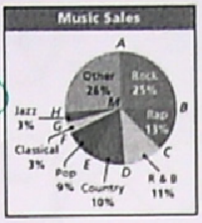
**1 Data Application**

The circle graph shows the types of music sold during one week at a music store. Find $m\widehat{BC}$.

$m\widehat{BC} = m\angle BMC$ (m of arc = m of central ∠)

$m\angle BMC = .13(360)$

$= 46.8°$

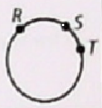



**CHECK IT OUT!** Use the graph to find each of the following.

1a. $m\angle FMC$ 1b. $m\widehat{AHB}$ 1c. $m\angle EMD$

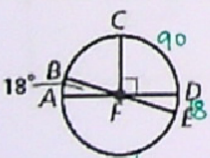
$.33(360) = 118.8°$   $.75(360) = 270°$   $.10(360) = 36°$

adjacent arcs are arcs of the same circle that intersect at exactly one point. $\widehat{RS}$ and $\widehat{ST}$ are adjacent arcs.




**2 Using the Arc Addition Postulate**

Find $m\widehat{CDE}$




$\boxed{108°}$

Find each measure.

2a. $m\widehat{JKL}$

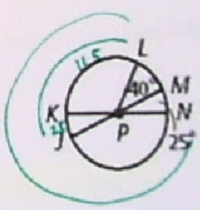
25 + 115

$\boxed{140°}$

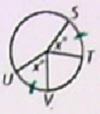
2b. $m\widehat{LJN}$

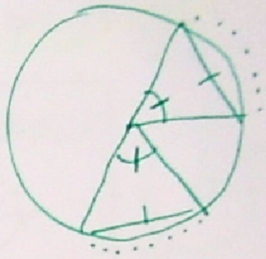
360
−40
−25

$\boxed{295°}$

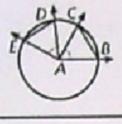
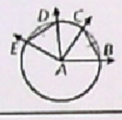


Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure, $\widehat{ST} \cong \widehat{UV}$.



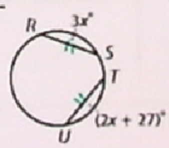
**Theorem 12-2-2**

| THEOREM | HYPOTHESIS | | CONCLUSION |
|---|---|---|---|
| In a circle or congruent circles: | | | |
| (1) Congruent central angles have ≅ chords |  | $\angle EAD \cong \angle BAC$ | $\overline{DE} \cong \overline{BC}$ |
| (2) Congruent chords have ≅ arcs |  | $\overline{ED} \cong \overline{BC}$ | $\widehat{DE} \cong \widehat{BC}$ |
| (3) Congruent arcs have congruent central <s |  | $\widehat{ED} \cong \widehat{BC}$ | $\angle DAE \cong \angle BAC$ |

## 3 Applying Congruent Angles, Arcs, and Chords

Find each measure.

**A** $\overline{RS} \cong \overline{TU}$. Find $m\widehat{RS}$.



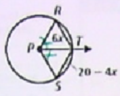

$3x = 2x + 27$

$x = 27$

$m\widehat{RS} = 81°$

Find each measure.

**3a.** $\overrightarrow{PT}$ bisects $\angle RPS$. Find $RT$.



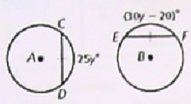

$6x = 20 - 4x$

$10x = 20$

$x = 2$

$RT = 12$

**3b.** $\odot A \cong \odot B$, and $\overline{CD} \cong \overline{EF}$. Find $m\widehat{CD}$.



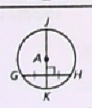

$25y = 30y - 20$

$-5y = -20$

$y = 4$

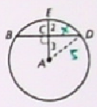
$25(4) = 100° = m\widehat{CD}$

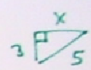
**Theorems**

| THEOREM | | HYPOTHESIS | CONCLUSION |
|---|---|---|---|
| 12-2-3 | In a circle, if a radius (or diameter) is perpendicular to chord then it bisects the chord and its arc. |  $\overline{CD} \perp \overline{EF}$ | $\overline{CD}$ bisects $\overline{EF}$ and $\widehat{EF}$. |
| 12-2-4 | In a circle the ⊥ bisector of a chord is a radius (or diameter). |  $\overline{JK}$ is ⊥ bisector of $\overline{GH}$. | $\overline{JK}$ is a diameter of $\odot A$. |

### 4 Using Radii and Chords

Find $BD$.



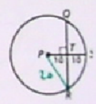

$x^2 + 9 = 25$

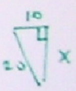
$x^2 = 16$

$x = 4$

$BD = 8$

**4.** Find $QR$ to the nearest tenth.




$QR = 34.6$

$x^2 + 100 = 400$

$x^2 = 300$

$x = \sqrt{300}$

$\approx 17.3$

2