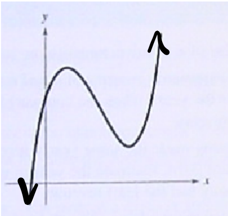
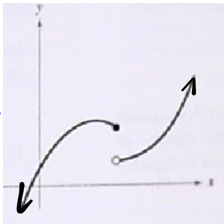
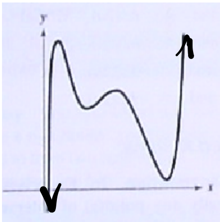
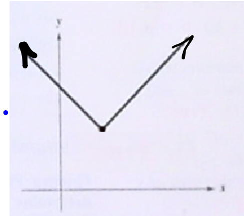
2.2 Polynomial Functions of Higher Degree

The graphs of polynomial functions are _______________.

Ex 1: Given the graphs below, which ones represent polynomial functions?

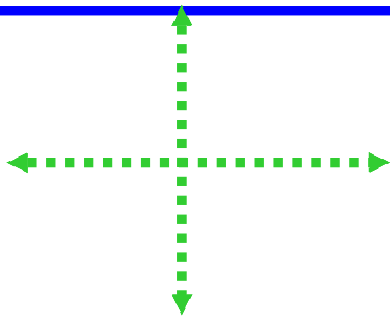
a.  b.  c.  d. 

Ex 2: Graph $f(x) = x^3$, find each:

domain: ______ increasing: _______

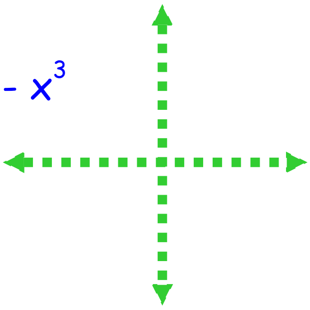
range: _______ even or odd? ______

intercept: _______ symmetry ________

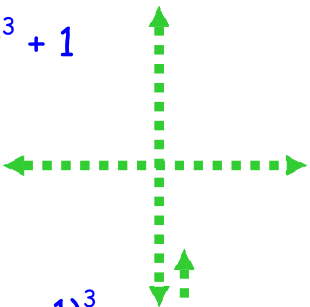


Ex 3: Sketch each:

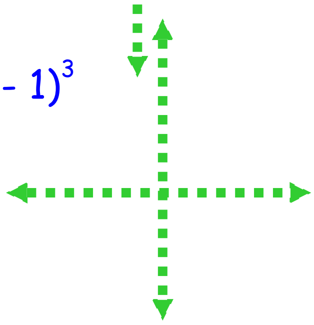
a. $g(x) = -x^3$



b. $h(x) = x^3 + 1$



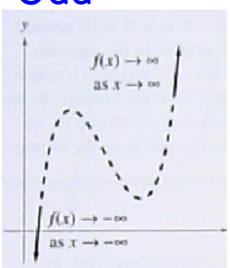
c. $k(x) = (x - 1)^3$

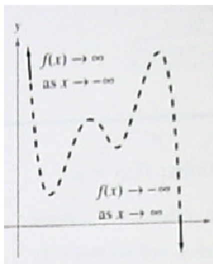


Leading Coefficient Test

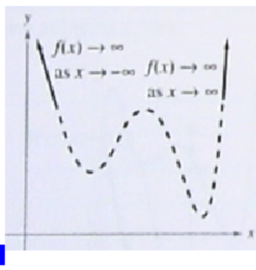
Given: $f(x) = a_nx^n + \ldots + a_1x + a_0$, where $a_n \neq 0$ then the graph of even and odd functions have the following behaviors:

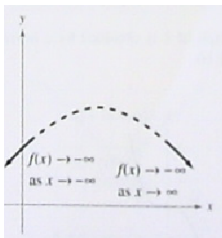
Odd





Even





Zeros of a Polynomial Function

Given a polynomial function of degree n, the following are true:

1. The function has at most ____ real zeros.
2. The graph has at most ______ relative extrema (relative min or relative max).

Real Zeros of a Polynomial Function

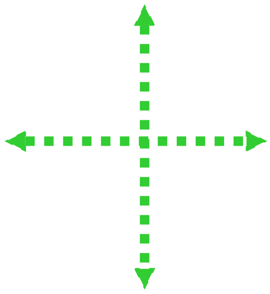
When f is a polynomial function and a is a real number, the following are true:

1. $x = a$ is a ______ of the function.
2. $x = a$ is a ________ when $f(x) = 0$.
3. $(x - a)$ is a _______ of $f(x)$.
4. $(a, 0)$ is an ____________ of the graph f.

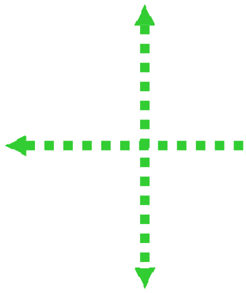
Ex 4: Find the zeros of $f(x) = x^3 - x^2 - 2x$

Algebraically

Graphically



Ex 5: Analyze: $f(x) = -2x^4 + 2x^2$

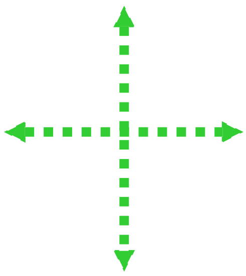


Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero $x = a$ of multiplicity k.

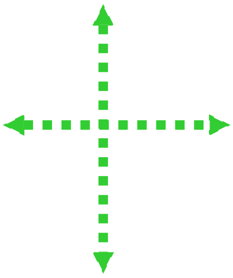
1. If k is odd, then the graph crosses the x-axis at $x = a$.

   Example: $f(x) = (x - 2)^3$

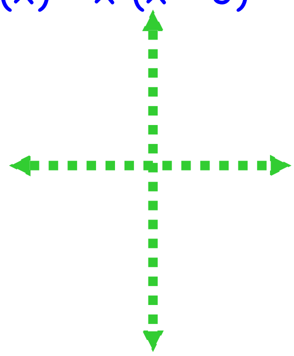


2. If k is even, then the graph touches (bounces) the x-axis at $x = a$.

   Example: $f(x) = (x - 2)^4$



Ex 6: Graph: $f(x) = x^2(x + 3)^3$



Ex 7: Find a polynomial function with:

zero at $-\frac{1}{2}$ mult 1

zero at 3 mult 2

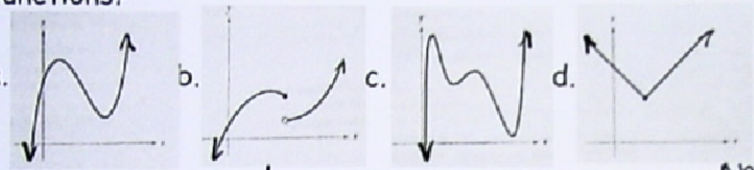
zero at 0 mult 3

2.2 Polynomial Functions of Higher Degree

The graphs of polynomial functions are continuous.

Ex 1: Given the graphs below, which ones represent polynomial functions?

a. b. c. d.



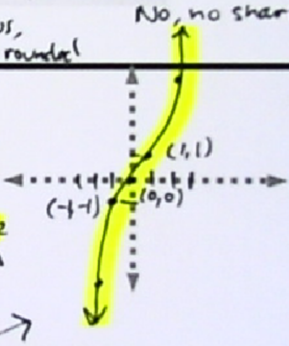
yes, continuous — No! not continuous, break — Yes, continuous, smooth and rounded — No, no sharp turns

Ex 2: Graph $f(x) = x^3$, find each:

domain: $(-\infty, \infty)$ increasing: $(-\infty, \infty)$

range: $(-\infty, \infty)$ even or odd? odd

intercept: (0,0) symmetry across the origin



x	f(x)
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27

→ use parent function $f(x) = x^3$ →

Ex 3: Sketch each: Leading Coefficient Test

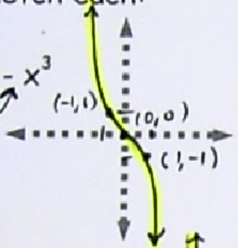
3 | 27

→ use parent function $f(x) = x^3$ →

Ex 3: Sketch each:

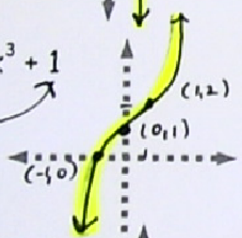
a. $g(x) = -x^3$

reflect across x-axis



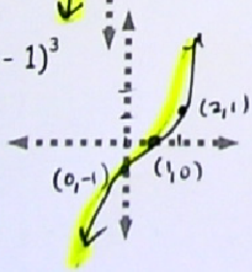
b. $h(x) = x^3 + 1$

vertical shift 1 unit up



c. $k(x) = (x - 1)^3$

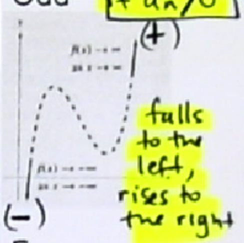
horizontal shift one unit to the right



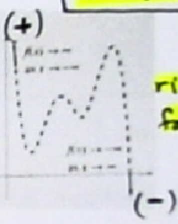
Leading Coefficient Test

Given: $f(x) = a_n x^n + \ldots + a_1 x + a_0$, where $a_n \neq 0$ then the graph of even and odd functions have the following behaviors:

Odd if $a_n > 0$ — if $a_n < 0$

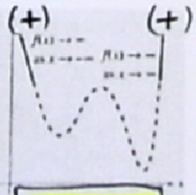


falls to the left, rises to the right

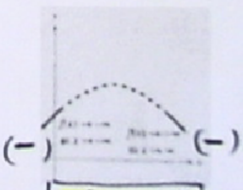


rises to the left, falls to the right

Even



if $a_n > 0$ graph rises on both left and right



if $a_n < 0$ graph falls on both left and right

Zeros of a Polynomial Function

Given a polynomial function of degree n, the following are true:

1. The function has at most **n** real zeros.
2. The graph has at most **(n-1)** relative extrema (relative min or relative max).

Real Zeros of a Polynomial Function

When f is a polynomial function and a is a real number, the following are true:

1. x = a is a **Zero** of the function.
2. x = a is a **Solution** when f(x) = 0.
3. (x - a) is a **factor** of f(x).
4. (a, 0) is an **x-intercept** of the graph f.

Ex 4: Find the zeros of $f(x) = x^3 - x^2 - 2x$

Algebraically

set $f(x) = 0$

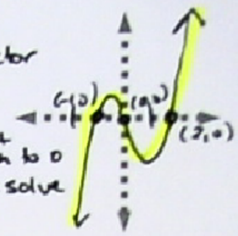
$0 = x^3 - x^2 - 2x$ factor

$0 = x(x^2 - x - 2)$

$0 = x(x-2)(x+1)$ set each to 0 then solve

Zeros $x = 0$, $x = 2$, $x = -1$

Graphically



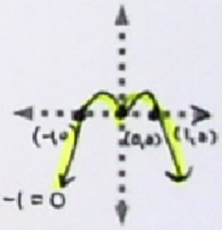
Ex 5: Analyze: $f(x) = -2x^4 + 2x^2$

$0 = -2x^4 + 2x^2$

$0 = -2x^2(x^2 - 1)$

$0 = -2x^2(x+1)(x-1)$

$-2x^2 = 0$	$x + 1 = 0$	$x - 1 = 0$
$x^2 = 0$	$x = -1$	$x = 1$
$x = 0$	mult 1 pass through	mult 1 pass through
mult 2 will bounce		



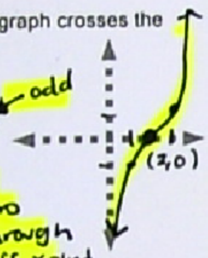
Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero x = a of multiplicity k.

1. If k is $\boxed{\text{odd,}}$ then the graph crosses the x-axis at x = a.

odd

Example: $f(x) = (x - 2)^3$



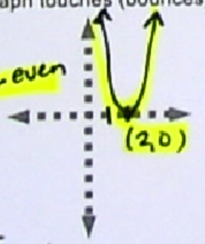
x = 2 is a zero

graph passes through this x-int.

2. If k is $\boxed{\text{even,}}$ then the graph touches (bounces) the x-axis at x = a.

even

Example: $f(x) = (x - 2)^4$



x = 2 is a zero

graph touches or bounces at this x-intercept

Ex 6: Graph: $f(x) = x^2(x + 3)^3$

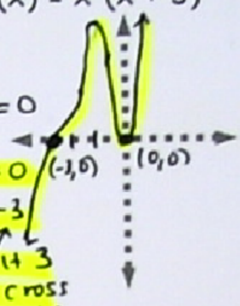
$0 = x^2(x+3)^3$

$x^2 = 0$ $\quad (x+3)^3 = 0$

$x = 0$ $\quad x + 3 = 0$

mult 2 (bounce) $\quad x = -3$

mult 3 will cross x-axis



Ex 7: Find a polynomial function

with: zero at $-\frac{1}{2}$ mult 1
zero at 3 mult 2
zero at 0 mult 3

final poly has leading power of 6

$$f(x) = (x + \tfrac{1}{2})^1 (x - 3)^2 (x - 0)^3$$
$$= (x + \tfrac{1}{2})(x-3)(x-3)x^3$$
$$= (x + \tfrac{1}{2})(x^2 - 6x + 9)x^3$$
$$= (x^3 - 6x^2 + 9x + \tfrac{1}{2}x^2 - 3x + \tfrac{9}{2})x^3$$
$$= x^3(x^3 - \tfrac{11}{2}x^2 + 6x + \tfrac{9}{2})$$

$$f(x) = x^6 - \tfrac{11}{2}x^5 + 6x^4 + \tfrac{9}{2}x^3$$