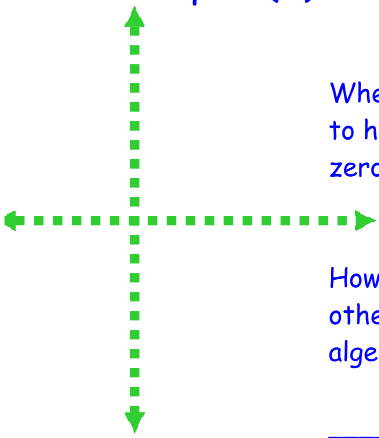
2.3a Real Zeros of Polynomial Functions

Ex 1: Graph $f(x) = 6x^3 - 19x^2 + 16x - 4$



Where does it appear to have a whole number zero? __________

How can we find the other two zero values algebraically?

____

$f(x) = (\qquad)^2\, q(x)$

Let's use long division to find q(x)!

Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, then use the result to factor the polynomial completely.

Ex 2: Divide the polynomial $3x^2 + 19x + 28$ by $x + 4$, then factor the polynomial completely.

Ex 3: Divide $8x^3 - 1$ by $2x - 1$

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}}\underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}$$

where $r(x) = 0$ *or* the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, then $d(x)$ *divides evenly* into $f(x)$.

The Division Algorithm can also be written as

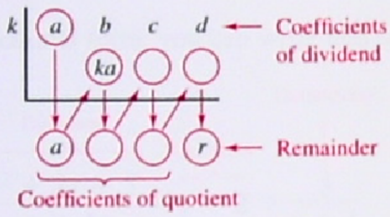
$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

Ex 4: Divide the polynomial $-2 + 3x - 5x^2 + 4x^3 + 2x^4$ by $x^2 + 2x - 3$

Now for the shortcut! Synthetic Division

Synthetic Division (of a Cubic Polynomial)

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern.



Vertical pattern: Add terms
Diagonal pattern: Multiply by k

Synthetic Division only works for divisors of the form $x - k$ also $x - (-k)$

Ex 5: Use synthetic division to divide:

a. $x^4 - 10x^2 - 2x + 4$ by $x + 3$

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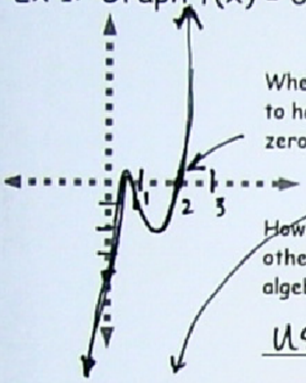

Where does it appear to have a whole number zero? $x = 2$ or $(2, 0)$

How can we find the other two zero values algebraically?

use long division

$f(x) = (x-2)^2 \cdot q(x)$

Let's use long division to find q(x)!

Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, then use the result to factor the polynomial completely.

$$\begin{array}{r l}
 & 6x^2 - 7x + 2 \\
x - 2 \,\big) & 6x^3 - 19x^2 + 16x - 4 \\
 & -6x^3 \pm 12x^2 \\
\hline
 & -7x^2 + 16x \\
 & \mp 7x^2 \mp 14x \\
\hline
 & 2x - 4 \\
 & -2x \pm 4 \\
\hline
 & 0
\end{array}$$

now factor $6x^2 - 7x + 2$

$(2x-1)(3x-2)$

Final factored polynomial $f(x) = (x-2)(2x-1)(3x-2)$

x-intercepts at $2, \frac{1}{2}, \frac{2}{3}$

Ex 2: Divide the polynomial $3x^2 + 19x + 28$ by $(x + 4)$, then factor the polynomial completely.

$$\begin{array}{r l}
 & 3x + 7 \\
x + 4 \,\big) & 3x^2 + 19x + 28 \\
 & -3x^2 + 12x \\
\hline
 & 7x + 28 \\
 & -7x + 28 \\
\hline
 & 0
\end{array}$$

factored polynomial $(3x+7)(x+4)$

Ex 3: Divide $8x^3 - 1$ by $2x - 1$

$$\begin{array}{r l}
 & 4x^2 + 2x + 1 \\
2x - 1 \,\big) & 8x^3 + 0x^2 + 0x - 1 \\
 & -8x^3 \pm 4x^2 \\
\hline
 & 4x^2 + 0x \\
 & -4x^2 \pm 2x \\
\hline
 & 2x - 1 \\
 & -2x \pm 1 \\
\hline
 & 0
\end{array}$$

☆ When the remainder is zero, that means the polynomial can be factored

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write in descending powers

$$\begin{array}{r|l} & 2x^2 \qquad\qquad +1 \\ x^2+2x-3 & 2x^4 + 4x^3 - 5x^2 + 3x - 2 \\ & -(2x^4 + 4x^3 - 6x^2) \\ \hline & \qquad\qquad x^2 + 3x - 2 \\ & \qquad\qquad -(x^2 + 2x - 3) \\ \hline & \qquad\qquad\qquad x + 1 \end{array}$$

remainder $r(x)$

Final answer

$$2x^2 + 1 + \frac{x+1}{x^2+2x-3}$$

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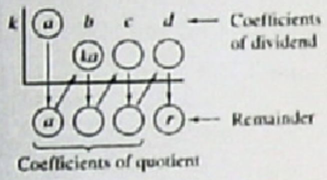

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$0x^3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & \downarrow & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array}$$

Quotient

remainder

$$1x^3 - 3x^2 - 1x + 1 + \frac{1}{x+3}$$

or

$$\boxed{x^3 - 3x^2 - x + 1 + \frac{1}{x+3}}$$