Pre-Calc 2.4b Complex Numbers

Writing quotients in standard form

To write the quotient a + bi and c + di in standard form, where c and d are not both zero, multiply and numerator and denominator by the complex conjugate of the denominator.

Example 1: $\dfrac{2+3i}{4-2i}$

Example 2: Perform the operation (Hint need LCD)

$$\frac{2i}{2+i}+\frac{5}{2-i}$$

Recall how to simplify a radical:

$$\sqrt{-24}=\sqrt{-1\cdot 4\cdot 6}=i(2)(\sqrt{6})=2\sqrt{6}i \quad \text{or} \quad 2i\sqrt{6}$$

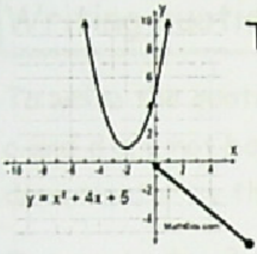
Example 3: Perform the operation and write in standard form.

(a) $\sqrt{-3}\sqrt{-12}$

(b) $\left(-1+\sqrt{-3}\right)^2$

Complex Solutions of Quadratic Equations

Recall that quadratics can have "imaginary solutions"



To solve - use the quadratic formula

Quadratic Formula:
For $ax^2 + bx + c = 0$,
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$x^2 + 4x + 5 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

no x-intercepts

no "real" solutions $\quad x = \dfrac{-4 \pm \sqrt{16-20}}{2} = \dfrac{-4 \pm \sqrt{-4}}{2} = \dfrac{-4 \pm 2i}{2}$

$$-\frac{4}{2} \pm \frac{2i}{2} = -2 \pm i$$

Example 4: Solve the quadratic equations below.

(a) $x^2 + 6x + 10 = 0$

(b) $\dfrac{7}{8}x^2 - \dfrac{3}{4}x + \dfrac{5}{16} = 0$

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Pre-Calc 2.4b Complex Numbers

Writing quotients in standard form

To write the quotient a + bi and c + di in standard form, where c and d are not both zero, multiply and numerator and denominator by the complex conjugate of the denominator.

Example 1: $\frac{2+3i}{4-2i}$

$= \left(\frac{2+3i}{4-2i}\right) \cdot \left(\frac{4+2i}{4+2i}\right)$ "FOIL"

$= \frac{8+4i+12i+6i^2}{16-4i^2} = \frac{2+16i}{20}$

$= \frac{2}{20} + \frac{16}{20}i = \boxed{\frac{1}{10} + \frac{4}{5}i}$

Example 2: Perform the operation (Hint need LCD) $\rightarrow (2+i)(2-i)$

$\frac{2i}{2+i} + \frac{5}{2-i} = \frac{2i(2-i)}{(2+i)(2-i)} + \frac{5(2+i)}{(2+i)(2-i)}$

$= \frac{4i - 2i^2 + 10 + 5i}{(2+i)(2-i)}$

$= \frac{12+9i}{4-i^2}$

$= \frac{12+9i}{5}$

$= \boxed{\frac{12}{5} + \frac{9}{5}i}$

Recall how to simplify a radical:

$\sqrt{-24} = \sqrt{-1 \cdot 4 \cdot 6} = i(2)(\sqrt{6}) = 2\sqrt{6}i$ or $2i\sqrt{6}$

Example 3: Perform the operation and write in standard form.

(a) $\sqrt{-3}\sqrt{-12} = \sqrt{3}i \cdot \sqrt{12}i = \sqrt{36}\,i^2 = 6i^2 = \boxed{-6}$

* Have to take out i before multiply!

(b) $(-1+\sqrt{-3})^2$

$= (-1)^2 - 2\sqrt{3}i + (\sqrt{3})^2(i^2)$

$= 1 - 2\sqrt{3}i + 3(-1)$

$= \boxed{-2 - 2\sqrt{3}i}$

Complex Solutions of Quadratic Equations

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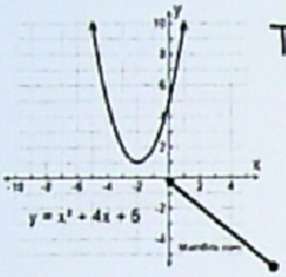

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no x-intercepts

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$$-\frac{4}{2} \pm \frac{2i}{2} = -2 \pm i$$

Example 4: Solve the quadratic equations below.

(a) $x^2 + 6x + 10 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36-40}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2i}{2}$$

$$= \boxed{-3 \pm i}$$

(b) $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$

Multiply both sides by 16!

$14x^2 - 12x + 5 = 0$

$$x = \frac{12 \pm \sqrt{(12)^2 - 4(14)(5)}}{2(14)}$$

$$x = \frac{12 \pm \sqrt{-136}}{28} \quad -1 \cdot 4 \cdot 34$$

$$x = \frac{12 \pm 2i\sqrt{34}}{28} = \boxed{\frac{3}{7} \pm \frac{\sqrt{34}}{14} i}$$

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